Modeling Sustainability Transitions on Complex Networks

There has been renewed interest in sociotechnical systems in the context of transitioning to a more sustainable society. While gains have been made in the qualitative understanding of sustainable transitions and sociotechnical systems, these approaches have not been well-operationalized. Given the importance of meeting future energy and environmental policy targets, there is need to develop predictive techniques and more robust methods to quantify and analyze sociotechnical systems undergoing rapid change and uncertainty due to sustainability pressures. Sustainability transitions depend on large-scale diffusion of technological and behavioral innovations on physical and virtual networked systems. Transitions can therefore be viewed as a subclass of diffusion phenomenon and subject to a range of mathematical and computational methods. We review, categorize, and critically assess methodological and theoretical approaches that integrate different aspects of sustainability, innovation, and complexity. We argue that these approaches should be adapted to improve our understanding of the behavior and dynamics of a broad range of sociotechnical systems to meet sustainability objectives. We therefore also make the conceptual link between complexity and sustainability as complimentary fields of research to inform policy and decision making to achieve more sustainable outcomes. © 2014 Wiley Periodicals, Inc. Complexity 19: 8–22, 2014

Key Words: complex networks; innovation diffusion; sustainability; sociotechnical systems; agent-based modeling

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1. INTRODUCTION

Understanding how sociotechnical systems might change and adapt over time under increasing physical and societal pressure is fundamental for transitioning to a more sustainable society. An important aspect of sustainability transitions is how new innovations in technology and human behavior spread throughout society. But many uncertainties exist over the scale, timing, and impacts of large-scale diffusion. For example, we do not know whether consumer demand will support the rapid uptake of advanced energy saving technologies to meet climate policy. This is based on our relative lack of understanding of the influencing feedbacks between human behavior and new technological advancements. Consequently, it is not well-understood how policy and strategy can influence the behavior and dynamics of innovation diffusion in sociotechnical systems, which underpins a broader sustainability transition. However, there is increasing awareness of the important role that complex networks can play in diffusion dynamics [1,2]. Global connectivity through virtual and physical infrastruc-

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ture has made large amounts of social and physical data available as never before. Complex network theory is becoming increasingly important for understanding that data and making sense of new found relationships [3]. Complexity sciences can therefore play a role in understanding the spread of new technological innovations that can influence adaptive behavior and sustainability objectives.

Complexity sciences are increasingly used to tackle issues related to sustainability including natural resource management and the tragedy of the commons [4,5], environmental security [6], and drawing parallels between evolutionary models and technological innovation [7,8]. However, that work differs from the broader framework of sustainability transitions that focuses on the coevolutionary processes between society and technology to meet sustainability objectives, largely influenced by environmental policy. There is increasing research on sustainability transitions and sociotechnical systems that draws heavily on diffusion phenomenon and innovation theory. However, much of the work is qualitative in nature and generally overlooks the important contributions that quantitative modeling has had on the field of diffusion. This article seeks to address this gap by tracing modeling developments, and arguing for the complementarity between the fields of complexity, innovation, and sustainability as a novel integration of theoretical models.

Relatively recent approaches have accounted for different aspects of sustainability and innovation in sociotechnical systems. One such approach is sociotechnical transitions, or transitions theory, which offers a broad qualitative framework for studying long-term societal and technical systemic change [9,10]. This work has largely built on Rogers [11] diffusion of innovation (DOI) theory. Similar to Rogers, these more recent approaches are also grounded in empirical historical examples, for instance transitions in transport technologies and the social, institutional, and technological factors that may have contributed to large-scale change [12]. Much of this literature has arisen from the policy agenda to transition to a more sustainable society, making the connection between the role of social and technological innovation and sustainability objectives [13].

However, because the transitions approach typically emphasizes long-term radical change operating at the decadal time-scale, it can miss short-range factors with potentially positive or negative consequences for stated policy objectives. For example, new understanding of diffusion processes on complex networks has shown that the short-term removal of highly connected nodes on physical and information networks can have unintended cascading consequences for overall system resilience and stability [14,15]. Because of the longer-time scales, the transition framework may also miss important incremental changes in the existing system that might have longer-term effects. For instance, short-term efficiency improvements in internal combustion vehicles could take market share away from adoption of alternative fuel vehicles (AFV) over the longer term [16]. This is due to the cumulative build up inherent in diffusion processes, which implies that short-term effects can have longer-term impacts on the system. We also know this to be true from Chaos theory where the initial conditions of a system can greatly influence longer-term trajectories that the system takes [17]. Moreover, due to its large scope (e.g., firms, consumers, technologies, legislation, and infrastructure) [18], the transitions framework runs the risk of being intractable, analogous to mathematical models with excessive parameters to ensure robustness at the expense of explanatory power.

Much of the sociotechnical transitions literature is also based on historical case examples, yet the scale of change that sustainability policy implies does not have a historical precedent. It is therefore not evident that historical case studies are effective for informing future policy and adaptive strategies under scenarios of massive social and environmental upheaval. Additionally, much of the literature on sustainability transitions has not accounted for our increasing understanding of the role that complex networks play in diffusion dynamics. And while one of the strengths of the sociotechnical or transitions framework is that it accounts for heterogeneity, this is often at the firm or market niche level [13] and does not generally capture individual decision making processes, strategic behavior, and personal preferences for technological attributes. DOI theory and many of the mathematical models subsequently developed have shown the important role of choice and preferences in adoption behavior [1,11].

In the context of understanding how a system changes over time, sustainability transitions can be viewed as a subclass of broader diffusion dynamics, which originally attempted to explain how new ideas, practices, and technologies spreads across societies, within, and between communities [19]. The field is interdisciplinary with roots in anthropology [11], economics [20], marketing [21], and mathematics [22] among others. Much of the original work was either adapted from, or influenced by epidemiology [22] where there was interest in how disease or other contagions spread throughout a population. Sustainability transitions are also influenced by innovation studies. There has been considerable research on a "theory" of innovation diffusion beginning with various early works by Floyd [23], Rogers [11], and Bass [21] along with newer contributions that build on the original theory [24–27]. Based on

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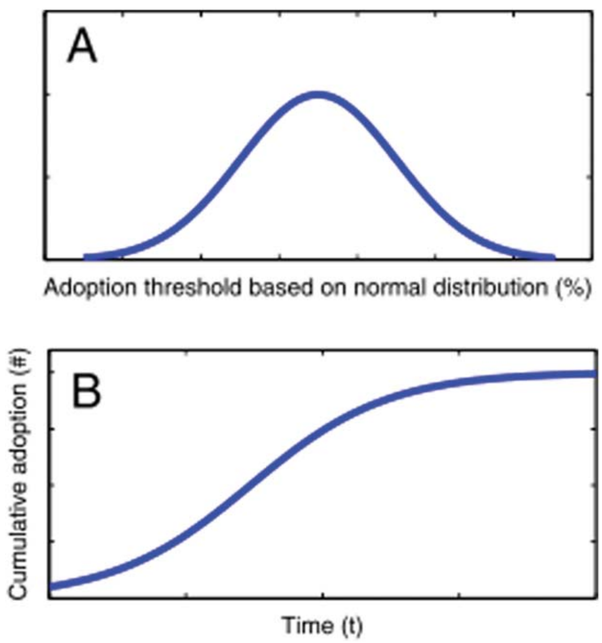
extensive empirical research, the premise is that new ideas and innovations largely spread through communication processes via interpersonal contact between people [11,21,24]. Consequently, diffusion processes such as innovations spreads through social dynamics. For instance, as an innovation becomes more widely accepted the social pressure to adopt exceeds each subsequent adopter category threshold level. Because individual thresholds for adoption are typically thought to be normally distributed (Figure 1A), cumulative adoption takes the form of a logistic sigmoidal curve of diffusion (Figure 1B). The bottom of the curve indicates a low level of initial adopters followed by a sudden acceleration in diffusion, and finally a leveling off reaching the top of the S-curve representing market saturation. This interpretation of diffusion suggests a mechanism behind the diffusion process and makes the important observation that individual level behavior (threshold effects) can result in system wide phenomenon (accelerated adoption leading to market saturation). This type of emergent systems level behavior arising from individual microlevel interactions is an early observation of what characterizes complex systems.

FIGURE 1



Conventional diffusion modelling based on (A) adoption threshold following a normal distribution leading to (B) cumulative adoption resulting in logistic S-curve showing slow growth, acceleration, and saturation effects over time (t). Based on empirical case studies, Rogers [11] suggests the following adopter categories showing the average percentile of each group within a population where, innovators (2.5%) are the first to adopt, followed by early adopters (13.5%), an early majority (34%), a late majority (34%), and finally laggards (16%). Those percentages are based on the normal distribution such that innovators are two standard deviations or more above the mean level of innovativeness.

From a theoretical and methodological perspective, there is considerable scope to apply new tools and techniques to further our understanding of the complex interactions between human behavior, innovation diffusion, and other dynamical processes in sociotechnical systems. The objective of this article is to review, categorize, and critically assess modeling theory and techniques that could be used to complement our current understanding of sustainability transitions and improve our ability to predict the behavior and dynamics of sociotechnical systems. This article proceeds with a broad categorization of theory at the dynamical systems and agent behavioral levels. We then discuss complementarity between sustainability, innovation, and complex networks as a way of integrating various methodological approaches, and close with implications for policy and future research.

2. MODELLING SYSTEM BEHAVIOR

2.1. Diffusion Models

In the natural sciences, the value of a theory has conventionally been tested against its' predictive ability [28]. In terms of an underlying mechanism, Rogers et al. [29] argues that innovations diffuse over time through communication channels among individuals interacting within a larger social system. Such empirical observations spurred interest in a more rigorous understanding of the underlying mechanisms by which diffusion processes occur. Mathematical models of diffusion processes were first developed in epidemiology to understand the spread of a disease and describe different states to which an individual within a population belongs. These deterministic models include the susceptible-infected-recovered (SIR), susceptible-infected (SI), and susceptible-infected-susceptible (SIS) among others [22,30]. Among the first of those models, which continues to be extended today [31,32] is the original SIR epidemiological Kermac–Mckendrick model [33] given by the following coupled nonlinear ordinary differential equations,

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$$\frac{dS}{dt} = -\beta SI \quad (1)$$

$$\frac{dI}{dt} = \beta SI - \gamma I \quad (2)$$

$$\frac{dR}{dt} = \gamma I \quad (3)$$

where $S(t)$ stands for the number of susceptible individuals, that is, individuals who do not get infected at time t; $I(t)$ represents the number of individuals who have been infected and are capable of spreading the disease; and $R(t)$ represents the number of individuals who have recovered from the disease. While β is the rate parameter that the infection spreads and γ is the rate parameter of recovery, which are both estimated. The key value governing the time evolution is the epidemiological threshold given by $R_o = \beta S_o/\gamma$ where S_o is the initial number of susceptible people. When $R_o < 1$ an individual who contracts the disease will infect less than one person and the outbreak will die out ($dI/dt < 1$). When $R_o > 1$ an infected person will infect more than one individual causing an epidemic ($dI/dt > 0$). Although this model was successful in describing many effects, it made simplifying assumptions. For instance, for a fixed population $N = S(t) + I(t) + R(t)$ all individuals in a population have equal probability of becoming infected at a rate β. Therefore, an individual can transmit the disease with βN others per unit time and the number of contacts between infected and susceptible is S/N. The number of new infections is then $\beta N(S/N)$ giving the rate of new infections as $\beta N(S/N)I = \beta SI$.

Advancements in network theory have since shown that preferential attachment to nodes or individuals often occurs on networks implying nonequal probability for the spreading phenomenon as a function of network structure. For example, a random graph G with N nodes where every pair of nodes are connected with a probability P results in a graph with approximately $pN(N-1)/2$ edges distributed randomly with most nodes assigned approximately the same number of edges following a Poisson distribution. However, empirical work has subsequently shown that the number of edges k connected to a node follows a power law distribution of the form $p_k \sim k^{-\gamma}$, representing a major break from previous theoretical work [34]. Nevertheless, despite the simplicity of early epidemiological models, they were effective at describing aggregate processes and subsequently applied to diverse social phenomenon such as the spread of rumours [35,36], information [37], and innovations [38,39]. Those early epidemiological models influenced to some degree the development of diffusion models specific to technological innovations mostly established in the marketing research literature since the 1960s, along with many new contributions from management and organizational sciences [21,24,25,40].

Innovation diffusion was first modeled at the aggregate level, simulating the penetration of a product into market characterized by the logistic curve and variants thereof [21,39,41]. More recent approaches have focused on individual adoption behavior based on preferences for product attributes [42–44]. Approaches that consider agent heterogeneity, uncertainty, and endogenous technological learning have also been explored [45,46] with crossover into complexity and network sciences in recent years [19,47–49]. The logistic curve which has been used to characterize diverse phenomenon originates from the natural sciences, but has since been applied to social systems across a range of settings from language acquisition [50] to vehicle ownership and energy infrastructure development [51]. The logistic curve originates from observations that many population growth processes begin exponentially shown by the ordinary differential equation,

$$\frac{dP(t)}{dt} = \alpha P(t) \quad (4)$$

where $P(t)$ is the population at time t and the growth rate is proportional to the population by some constant α, and the growth rate at time t is defined as the derivative $dP(t)/dt$. If the population is P_0 at time t_0, then $P(t)$ satisfies the initial value problem $dP(t)/dt = \alpha P(t)$, $P(t_0) = P_0$. This differential equation can be solved by the following function,

$$P(t) = \beta e^{\alpha t} \quad (5)$$

where e is the base of the natural logarithm, α is the growth rate constant, and β is the initial population at $P(0)$. The growth rate constant α is often expressed as a percent (%) where $\alpha = 0.02$ implies the population P grows continuously at 2% per time unit. Although the onset of many growth processes begins exponentially, natural and social systems cannot sustain exponential growth indefinitely because of negative feedback mechanisms or signals sent from the system that slows growth. This "limits to growth" was first described in the context of a physical system's carrying capacity that places an upper limit on unbounded growth due to finite resources, which was the precursor to broad notions of sustainability. This understanding subsequently led to policy initiatives to improve efficiency across the entire lifecycle from resource extraction to production to end-use. To model this upper limit, variations on the exponential growth model can be used with the most popular being the logistic equation introduced by Verhulst [52] and popularized in mathematical biology by Lotka [53]. The logistic equation is,

$$\frac{dP(t)}{dt} = \alpha P(t)\left(1 - \frac{P(t)}{K}\right) \quad (6)$$

which retains the population $P(t)$ and growth rate constant α from the exponential model, but now adds a negative feedback term $(1 - \frac{P(t)}{K})$ that slows

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the growth rate of a population as the limit K is approached. The feedback term $(1-\frac{P(t)}{K})$ approaches one when $P(t) << K$ and approaches zero as $P(t) \rightarrow K$. Therefore, the growth rate begins exponentially, but then decreases to zero as the population approaches the limit K resulting in a sigmoidal growth trajectory. The logistic equation can be solved by the following function,

$$P(t)=\frac{K}{1+e^{-\alpha(-\beta t)}} \tag{7}$$

Three parameters α, β, and K are needed to specify the S-shaped curve of Eq. (7). The growth rate parameter α specifies the steepness of the sigmoidal curve. The parameter β specifies the time when the curve reaches $\frac{1}{2} * K$ or the midpoint of the growth trajectory labeled t_m. The parameter K is the asymptotic limit that the growth curve reaches, that is, physical carrying capacity or market saturation. Although the logistic function is a simple mathematical model, it displays nondeterministic chaotic properties when found in its recurrence form known as the logistic map given by,

$$X_{t+1}=rX_t(1-X_t) \tag{8}$$

where X_t is a number between zero and one representing the ratio of an existing population to the maximum possible population in year t, and r is a positive number representing the combined rate for growth and decay of a population. For practical applications, X has to remain on the interval $0<X<1$. If X exceeds unity, further iterations diverge toward $-\infty$ where the population becomes extinct. When $F(X)$ of the logistic map attains a value of $r/4$ at $X=\frac{1}{2}$ the equation attains nontrivial dynamical behavior when $r<4$. However, if $r<1$ all trajectories are attracted to $X=0$. Therefore, nontrivial dynamical behavior requires $1<r<4$ otherwise the population becomes extinct [54]. The key point is that although the model is mathematically simple, it has proven widely useful in approximating many real world processes because it captures the negative feedback found in natural systems that can result in nonequilibrium dynamics, a key characteristic of natural and social systems.

2.2. Dynamics of Innovation

We can now see the direct influence that observations of limits to growth in physical systems and related dynamical systems models have had on understanding the processes of innovation diffusion in technological and social systems. For instance, at the macrolevel, a consistent finding of innovation diffusion has been the cumulative pattern of adoption characterized by the same logistic function,

$$y_t=b_0+\frac{1}{1+e^{-b1t}} \tag{9}$$

The parameter y_t is now reinterpreted as the proportion of adopters at time t, the previous number of adopters is b_0, and the adoption rate parameter to be estimated is b_1. We can see that the estimated parameters have been simplified in this interpretation since, the number one in Eq. (9) represents 100% share of the market, that is, the "carrying capacity" that ultimately limits the amount of adoption. The sigmoidal curve is now interpreted as a slow rate of adoption in the initial phases of diffusion followed by an accelerated take off, but eventually reaching a market saturation effect. Although the logistic model can be used to compare growth rates for different innovations, it is limited in its applicability because it does not capture many dynamics found in real markets [1]. A considerable improvement, and arguably the most cited mathematical innovation diffusion model can be traced back to the Bass model [21],

$$\frac{dn(t)}{dt}=[M-n(t)]\left[p+\frac{q}{M}n(t)\right], n(0)=0 \tag{10}$$

where $n(t)$ is the number of individuals that adopted the product at time t and M is the population size. The parameter p (coefficient of innovation) describes the likelihood of an individual to adopt from external influences such as mass media, and q (coefficient of imitation) captures internal influences on adoption, such as the influence of other individuals who have already adopted. Relating innovation diffusion to the spreading of an epidemic, imitation is viewed as a contagion effect. For pure innovation ($p>0$, $q=0$), diffusion follows a modified exponential curve; for pure imitation ($p=0$, $q>0$), diffusion is logistic. Other properties are $(p+q)$ controls scale, (q/p) controls shape, and $(q/p)>1$ is required to obtain an S-curve [27]. This was found to be a considerable improvement from the one parameter interpretation of the logistic equation in accounting for multiple influences on diffusion. For instance, as individuals decide to adopt based on the term $p+q(n/M)$, the relationship between external and internal influences and how their combined effects influence individual behavior is made explicit. The model succeeds to some degree in succinctly capturing both system level and individual level behavior described by Roger's theory. The Bass model can be solved by the following function,

$$F(t)=\frac{1-e^{-(p+q)t}}{1+(\frac{q}{p})e^{-(p+q)t}} \tag{11}$$

where $F(t)=n(t)/M$ is the fraction of adopters at time t. Because the model also incorporates the percentage of adopters at each time point, it provides an estimate of the growth attributed to personal network influence. Empirically, the Bass model captures the S-curve for cumulative adoption of various products. Extensive empirical application of the model found typical values for the parameters to be $p=0.03$/year and $q=0.38$/year, with $p<0.01$/year and q typically in the range 0.3–0.5/year [48].

The model has been widely used for forecasting the market penetration of durable goods, and estimating the

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rate of diffusion attributed to different theoretical aspects of external influence p and internal influence q [24]. Although the Bass model has proven widely successful, we can see that the factors that influence adoption are aggregated into single parameters p and q. Additionally, the rate of new adoptions is equal to $(q/M)(M-n)$ based on the assumption that each of the $(M-n)$ nonadopters can be influenced by all n adopters. This implicitly assumes that all agents are fully connected to each other, which does not reflect real-world networks. Many derivations of the Bass model have since been developed with additional parameters to better account for consumer and market heterogeneity thus attempting to disaggregate some of the assumptions inherent in the original model. Important extensions were made to account for specific attribute effects such as price [55] given by

$$h(t) = (\beta_0 + \beta_1 F(t))\exp(-\beta_2 P(t)) \tag{12}$$

where $P(t)$ is a price index with $P(0) = 1$. This was used to examine pricing strategies showing high prices decreasing with diffusion can be a poor strategy compared to an optimal constant price, or where the price starts low, peaks, and then falls. Further work examined external advertising effects with the following modification

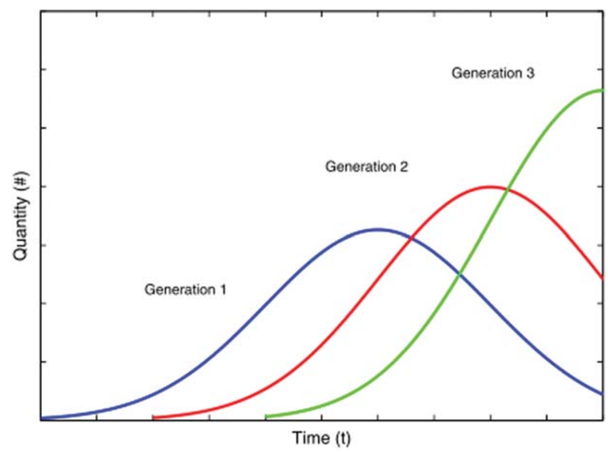
$$h(t) = (\beta_0 + \beta_1 F(t) + \beta_2 \ln(A(t))) \tag{13}$$

where $A(t)$ is expenditure on advertising at time t, and coefficients β_0 is publicity, β_1 is word of mouth effects, and β_2 is advertising [56]. General functional forms were further developed allowing for price effects on the probability of adoption and market potential such as,

$$f(t)=(p+qF(t))P^{\alpha}(t)(H(t)P^{\beta}(t)-F(t)) \tag{14}$$

where $H(t)$ is the proportion of households with access to new technologies. It was found that price-affected adoption probability $(\alpha<0,\ \beta=0)$ for relatively high-priced technologies [57]. DOIs via contagion through social networks often do not account for the higher price effects associated with sustainable technologies. Models that can incorporate price effects and social influence are important for analyzing the adoption of sustainable technologies that have to compete against a lower-priced incumbent technological system.

FIGURE 2



The Norton Bass Model showing successive technological generations (G) showing superior technology (G_{t+1}) displacing earlier technology (G_t).

Improvements have also been made to better capture the multigenerational dynamics of technological diffusion. For instance, an important empirical observation was that the innovation diffusion process was often comprised of multiple diffusion processes that can occur simultaneously or overlap (Figure 2). An important extension of the Bass model was to capture multiple generations of innovation diffusion introducing additional time dynamics as follows,

$$S_{1,t} = F(t_1)m_1[1 - F(t_2)], \tag{15}$$

$$S_{2,t} = F(t_2)[m_2 + F(t_1)m_1][1 - F(t_3)], \tag{16}$$

$$S_{3,t} = \{m_3 + F(t_2)[m_2 + F(t_1)m_1]\}, \tag{17}$$

where $m_i = a_i M_i$ and M_i is the incremental number of ultimate adopters of the ith generation product, a_i is the average continuous repeat buying rate among adopters of the ith generation product, and t_i is the time since the introduction of the ith generation product, and $F(t_i) = [1 - e^{-(p+q)ti}]/[1 + (q/p)e^{-(p+q)ti}]$ [58]. This was an important extension of the model because it was now able to capture the diffusion of multiple generations of innovations characterized by the phasing in and out of competing technologies.

2.3. Critical Discussion

Although macro mathematical models can capture some elements of internal and external influences on adoption, there needs to be continued improvement in the predictive ability and explanatory power of models that can account for the dynamics and uncertainty of future conditions. This is important because we are interested in multiple diffusion processes occurring simultaneously such as how new sustainability innovations have to compete against the incumbent

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system now and into the future. This has important policy implications in terms of incentivizing the phase out of undesirable technologies or accelerating the adoption of beneficial ones. Furthermore, the analysis needs to go beyond the adoption and diffusion of technology, but ultimately to understand how the technology is used and the emergent patterns that might arise. End-use will be a function of agent preferences, strategic behavior, and other microlevel influencing factors, which will impact the performance of the larger system, for example, energy end-use patterns determines generation capacity and the resulting carbon emissions from the power system. This calls for an understanding of technological performance and use across scales from the individual to the larger system.

New computational techniques that can capture agent heterogeneity can also inform mathematical modeling approaches. For example, the Bass model and its' many derivatives are aggregate models that describe the behavior of entire populations. There have since been considerable developments for modeling individual level interactions and how that can affect aggregate diffusion processes [48]. Although macrolevel models can become increasingly robust with the inclusion of additional parameters, by definition, they are best suited for describing the average behavior of a system. This point can be forgotten as it is often implied that the more parameters in a model, the better it is at capturing real-world phenomenon, when often times a simple model that captures the salient features of a system may have far better explanatory power [59].

3. MODELING AGENT BEHAVIOR

3.1. Game-Theoretic Models

Macrolevel models typically based on differential equations have been widely successful in describing and in some instances forecasting system level behavior. Yet, one of the important drawbacks is their inability to capture disaggregated behavior that may give rise to emergent phenomenon. Such emergent properties are a defining characteristic of complex systems with important implications for diffusion phenomenon. Many models have been developed to assess individual behavior but only a few are formulated as mathematical relations [60].

A prominent approach is game theory, which is the concept of strategic success based on cooperation and competition among individuals [61]. A central appeal to game theory is that it has a set of well-developed analytical tools that have explanatory and predictive power for a broad class of strategic situations that involve interactions between numerous decision makers. Game theory assumes that players form rational beliefs about what other players in the game will do, and then chooses a response that will maximize its payoff. This poses limiting assumptions on human behavior because it is assumed that players have well-defined goals and preferences that can be described by a specified utility function, where the player's goal is to achieve an outcome from a game that maximizes their utility. Despite these limiting assumptions, the mathematical structure of game theory has allowed for abstraction away from specific problem domains providing a consistent methodology for assessing a broad class of agent behavior in a variety of situations.

The representation of a game specifies the players, feasible actions (pure strategies), and the payoffs received for each possible combination of actions (strategy profile) that could be chosen by the players. A game can be formally denoted as $G=\{L_1,\ldots,L_N;\ \ u_1,\ldots,u_N\}$ assuming a static, one-shot game, simultaneous decision making with complete information. Let $n=1,\ldots,N$ denote the players; $L_n=\{e_{n1},\ e_{n2},\ldots e_{nQ}\}$ the set of pure strategies available to player n, with $s_n \in L_n$ an arbitrary element of this set; $(s_1,\ldots,s_N)$ is a given strategy profile of all players; and, $u_n(s_1,\ldots,s_N)$ is player n's payoff function, which is the measure of satisfaction if the strategy profile $(s_1,\ldots,s_N)$ is achieved. For two player games $n=1, 2$ and available strategies are discrete $L_1 = \{e_1,\ldots,e_Q\}$, $L_2=\{e_1,\ldots,e_R\}$ the game is typically represented in a bimatrix form $G=(\boldsymbol{A}, \boldsymbol{B}^T)$.

Importantly, there are also N-player games where $N>2$, which is more representative of real-world situations. For example, an important precursor to sustainability are issues around resource management and the tragedy of the commons [62] which can be viewed as simultaneous N-player game. Consider a strategy for rancher N is to choose a number of livestock represented by a nonnegative real number $g_n \in [0,\ \infty]$ to graze on an open pasture. A single livestock implies a cost c and a benefit $v(G)$, which is a function of all livestock $G=g_1+\cdots+g_N$ grazing on the pasture. However, once the pasture becomes degraded, the value of the livestock decreases from malnutrition. Assuming $v'(G)<0$ and $v''(G)<0$, the rancher's payoff is

$$u_n = [v(G) - c]g_n \tag{18}$$

Rational game theory predicts emergent behavior to be a Nash equilibrium (NE) where a strategy profile $s^*=(S_1^*, \ldots, S_N^*)$ is a NE, if

$$\forall n, \forall S_n \neq S_n^* : u_n(S_n^*, S_{-n}^*) \geq u_n(S_n^*, S_{-n}^*) \tag{19}$$

Where each agent's strategy S_n^* is a best response to the strategies of all other players [63]. This means no rancher has unilateral incentive to change strategy, which results in overgrazing, and the NE is not Pareto efficient. A partial solution to the problem is for a central planner to fix the total number of livestock G allowed on the pasture [64]. From a game theoretic perspective, international cooperation

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on reducing greenhouse gas emissions suffers from similar incentives to free ride without intervention from a central planner, or in this case international agreement on emission levels [65]. Assessing these forms of inefficiencies is known as Public Good games, which are applicable to a large class of sustainability issues involving resource use and planning with multiple competing interests.

Multiplayer games are increasingly used to tackle issues in complexity involving agent behavior on networks [64]. The underlying premise in a networked games model is that agents' payoffs depend on their own actions and the actions of their neighbors determined by the network of connections. Galeotti et al. [66] show that under complete information there is no relation between network structure and individual pay off, but under incomplete information, networks can influence individual behavior. Consider a finite set of agents $N=\{1,2,\ldots,n\}$ connected in a matrix network $g \in \{0,1\}^{nXn}$, with $g_{ij}=1$ implying i's payoff is influenced by j's strategy. Setting $g_{ii}=0$ for all $i \in N$, and $N_i(g)=\{j|g_{ij}=1\}$ is the set of neighbors of i. The degree of player i, $k_i(g)$ is the number of i's connections giving

$$k_i(g)=|N_i(g)| \tag{20}$$

If each player i's strategy is x_i in X, in a subset [0,1] assuming 0, 1 $\in X$, the payoff of player i is

$$u_{k_i(g)}(X_i, X_{N_i(g)}) \tag{21}$$

when the action profile is $x=(x_1, \ldots, x_n)$ and $X_{N_i(g)}$ is the profile of actions taken by neighbors of i in the network g. Empirical work has shown that in certain situations agents' gain more when selecting an action that has also been taken by many other players (strategic complementarity) such as the adoption of a new technology, whereas in other contexts the payoff can decrease (strategic substitutability) such as contribution to a public good. A networked game can analyze both these situations. Consider a payoff that depends on the sum of actions, where player i's payoff function when selecting x_i and neighbors k choose the action profile $(x_1,\ldots,x_k)$ is:

$$u_i(x_i, x_1, \ldots, x_k)=f(x_i+\lambda\sum_{j=1}^{k} x_j)-c(x_i) \tag{22}$$

where $f(\cdot)$ is nondecreasing and $c(\cdot)$ is a cost function associated with own effort. The parameter $\lambda \in \mathbb{R}$ determines the externality across players' actions. The shape and sign of λf specifies the effects of neighbors' action choices on one's own optimal choice. This gives strict strategic substitutability if differentiability $\lambda f'$ is negative and strategic complementarity if positive. A particular case is a best shot public goods game where there are spill over's between players' actions. Consider an action profile $X=\{0, 1\}$ where action 1 means acquiring new information, where $f(0)=0$, $f(x)=1$ for all $x \geq 1$. Costs satisfy $0=c(0)<c(1)<1$ resulting in a game of strategic substitutes and positive externalities [66,67]. Such a model would have broad applicability for strategic decision making and sustainability transitions, such as the accelerated diffusion of new low-carbon technologies through positive knowledge transmission between agents or firms. Conversely, the model could also assess strategic situations where the potential benefits outweigh the costs of phasing out harmful technologies or behavior over a specified time frame.

3.2. Decision-Theoretic Models

Another important approach related to decision theory is choice modeling, which is based on individual preferences for different alternatives [42]. For the disaggregated analysis of socio-technical systems, these models can be useful because they are able to explicitly link individual preferences with technological attributes and are focused on how individuals choose among different competing options. This has important implications for diffusion of new sustainability innovations that have to compete against incumbent technologies. Although there have been long standing approaches for assessing individual level behavior, choice modeling has been the standard for evaluating decision making. These models are usually derived under an assumption of utility maximizing behavior by the decision maker. Models derived in this way are more generally known as random utility models (RUMs) where an agent or decision maker n has a choice among J alternatives. The utility that decision maker n obtains from alternative j is $U_{nj}=1,\ldots,J$. The behavioral model is then choose alternative i if $U_{ni}>U_{nj}\ \forall j \neq i$. Now consider that we do not observe the agent's utility but can observe specific attributes $x_{nj}\ \forall j$ of the alternatives j faced by the decision maker along with characteristics of the decision maker s_n, where we can specify a function $V_{nj}=V(x_{nj}, s_n)\ \forall j$, called representative utility that relates those observed factors to the decision maker's utility. However, there are aspects of utility that we do not observe $V_{nj} \neq U_{nj}$ such that utility can be decomposed into,

$$U_{nj}=V_{nj}+\epsilon_{nj}, \tag{23}$$

where, ε_{nj} captures the factors that affect utility but are not included in V_{nj}. We do not know, $\varepsilon_{nj}\ \forall\ j$ and therefore treat these terms as random. The joint density of the random vector ε_{nj} is denoted $f(\varepsilon_n)$. We can therefore make probabilistic inferences about the decision maker's choice such that the probability that decision maker n chooses alternative i is

$$P_{ni}=\text{Prob}\,(U_{ni}>U_{nj}\ \ \forall j \neq i), \tag{24}$$

$$\text{Prob}\,(V_{ni}+\epsilon_{ni}>V_{nj}+\epsilon_{nj}\ \ \forall j \neq i), \tag{25}$$

$$=\text{Prob}\,(\epsilon_{nj}-\epsilon_{ni}<V_{ni}-V_{nj}\ \ \forall j \neq i). \tag{26}$$

The probability is a cumulative distribution where each random term $\varepsilon_{nj}-\varepsilon_{ni}$ is below the observed quantity

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$V_{ni} - V_{nj}$. With the density $f(\varepsilon_n)$, the cumulative probability is,

$$P_{ni} = \text{Prob}\,(\epsilon_{nj} - \epsilon_{ni} < V_{ni} - V_{nj} \forall j \neq i), \tag{27}$$

$$= \int_{\epsilon} I(\epsilon_{nj} - \epsilon_{ni} < V_{ni} - V_{nj} \;\forall j \neq i) f(\epsilon_n) d\epsilon_n, \tag{28}$$

where $I(\cdot)$ is the indicator function, which equals 1 if true and 0 if false. Different choice models are obtained from specifying different densities for $f(\varepsilon_n)$ [68]. The multidimensional integral in Eq. (28) only takes a closed form for certain specifications of $f(\cdot)$. The most popular and widely used choice model is the multinomial logit model (MNL) derived by McFadden [42] showing that the unobserved portion of utility is distributed extreme value. When representative utility is specified to be linear in parameters, the model is given by,

$$P_{ni} = \frac{e^{\beta' X_{ni}}}{\sum_{j=1}^{J} e^{\beta' X_{nj}}} \tag{29}$$

where the probability P_{ni} of agent n selecting alternative i is a function of a vector of observable attributes X_{ni} of alternative i observed by agent n and a vector of random coefficients β representing the agent's preferences or tastes for those attributes. Although the MNL has been the standard for many years, it is restricted by its independence of irrelevant alternatives (IIA) assumption, which can unrealistically predict that a change in the attributes of one alternative changes the probabilities of the other alternatives proportionately [42,69]. In recent years, advances in computational techniques have allowed for development of more flexible derivations such as the mixed logit (ML) that relaxes the IIA assumption and is able to accommodate general patterns of competitiveness and heterogeneity across individuals in their sensitivity to exogenous variables [68,70]. McFadden and Train [71] have shown that the ML can approximate any underlying RUM. The ML integrates the multinomial logit formula over a distribution of unobserved random parameters as follows.

$$P_{ni}(\theta) = \int_{-\infty}^{+\infty} L_{ni}(\beta) f(\beta|\theta) d(\beta),$$
$$L_{ni}(\beta) = \exp\,(\beta' x_{ni}) / \sum_j \exp\,(\beta' x_{ni}) \tag{30}$$

where P_{ni} is the probability that agent n chooses alternative i. The vector x_{ni} are observed variables specific to individual n and alternative i, β represents parameters which are random realizations from a density function $f(\cdot)$, and theta θ is a vector of underlying moment parameters characterizing $f(\cdot)$. Monte Carlo simulations are used to numerically integrate Eq. (30), which computes the integrand at a sequence of random points and calculates the average integrand values. The basic principle is to replace a continuous average by a discrete average over randomly chosen points and by the strong law of large numbers achieve approximate convergence [70]. The flexibility of the ML allows simulation in scenario frameworks where preferential behavior can be specified for specific innovation attributes. This allows us to assess adoption patterns under different future scenario conditions influenced by changing consumer behavior and evolving technological performance, which are key underlying drivers for sustainability transitions.

3.3. Critical Discussion

One of the central criticisms of behavioral models is the assumption of utility maximizing behavior where agents act with perfect foresight and rationality. Much empirical work has been done by Kahneman and Tversky [72] to demonstrate the bounded rationality of agents. Those are important criticisms to be sure, but in many instances are applicable to a sub set of decision making that involves high risk and monetary loss. Moreover, those approaches are not yet well-developed for modeling purposes. In terms of simulation, the latest computational advancements in behavioral models offer a quantitative framework to understand decision behavior across wide ranging phenomenon. Also some behavioral models can be interpreted as simply describing the relation of explanatory variables to the outcome of a choice situation without necessarily implying a utility maximizing motivation behind the choice [68]. Nevertheless, greater understanding of different behavioral approaches will complement each other in forming a more complete understanding of the variety and nuances of human decision making ultimately leading to a more comprehensive theory [45]. For instance, important advancements have been made in complex systems for modeling bounded rational behavior through agent-based simulations, which we discuss below.

4. SOCIOTECHNICAL DYNAMICS ON NETWORKS

4.1. Heterogeneous Networks

There is complementarity between theoretical models of sustainability, innovation, and complexity to improve our understanding of the behavior and dynamics of sociotechnical systems. Complexity science in particular has become increasingly applied to myriad issues arising in politics, economics, engineering, and environmental sciences, because it provides theoretical tools and frameworks to understand and influence how systems change over time [73]. Although complexity does not yet have an overarching framework, there is increasing convergence around network science, because complex systems are composed of interconnected components. The study of networks from both mathematical and sociological perspectives has been around for many years. However, there has been a resurgence of network analysis in

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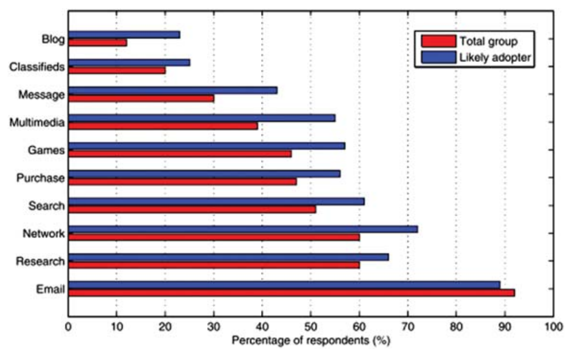
FIGURE 3



Online activities between total group surveyed and likely adopter of an electric vehicle. $N = 1046$; likely adopter group = 37.2% of total. On line activities that had the highest frequency were email (92.1%), researching topics of interest (60.3%), social networking (60.1%), search (51.3%), and purchasing products (47.4%). Among the likely group, email (89.4%), social networking (71.5%), research topics (66.0%), search (61.4%), and online games (57.4%) were the top five online activities [76].

recent years, largely due to increasing virtual and physical interconnectivity through computers and communication networks around the world [74]. Network science therefore has important implications for understanding the spread of information, technology, and behavioral phenomenon occurring on diverse network typologies. Figure 3 shows the array of online activities of a potential adopter group for new energy innovations. The three highest used categories involve communications and information, key drivers behind the spread of ideas, and innovations through information networks [75].

There is increasing recognition of the influence of network structure on spreading phenomenon in general [77] and innovation diffusion in particular [49]. Although early spatial autocorrelated methods accounted for the spatial effects on diffusion processes, they did not account for individual positioning within a network, and overall network topology. Early models of influence on individual adoption within a network from nonrandom mixing can be captured by an exposure function,

$$E_i = \frac{\sum w_{ij} y_i}{\sum w_i} \quad (31)$$

where E_i is network exposure, w is a social network weight matrix broadly defined, and y is a vector of adoptions. This could account for a variety of influences including information transmission, persuasion, pressure, or other social influences such as, the degree of structural equivalence, which is a measure of similarity in network position [19]. Exposure E_i can also be weighted to reflect other network properties such as centrality to reflect influence by opinion leaders, or weighted based on distance between network ties. However, only relatively recently have we come to discover the high level of heterogeneity in many real-world networks. In contrast to a homogenous network, where nodes i have a typical degree k close to the average $\langle k \rangle$, many real-world networks display hierarchy where a few nodes have a high degree and the remaining majority have low degrees. This relationship is characterized by a heavy-tailed degree distribution approximated by a power law $P(k) \sim k^{-\gamma}$ [34]. This model shows that when nodes were added they displayed preferential behavior for attaching to certain other nodes with a probability proportional to the degree of the target node. Other key characteristics of network heterogeneity are high clustering coefficients $C(p)$ where if node i has k connections then at most $k(k-1)/2$ edges exist between its neighbors (local interactions); and low average path lengths $L(p)$ defined as the number of edges in the shortest path between two nodes averaged over all pairs of nodes (global interactions) [78]. That handful of properties fundamentally altered our understanding of empirical networks and much work has since been developed to account for network heterogeneity.

Analytical solutions to dynamical processes on heterogeneous networks have proven elusive; therefore, mean-field techniques that account for varying connectivity patterns have been developed known as heterogeneous mean-field (HMF) approaches [2]. One example, is a reformulation of the SIS model where nodes are grouped in the same degree class k and i_k and s_k are, respectively, the fraction of nodes with degree k in the infected (I) and susceptible (S) class given by the following differential equation,

$$\frac{di_k(t)}{dt} = -\mu i_k + \lambda[1 - i_k(t)]k\phi_k(t) \quad (32)$$

where μ is the recovery rate, λ is the probability of transmission, and ϕ_k is the density of infected neighbors with nodes of degree k, which is the probability of contacting an infected individual. The average probability that any neighbor of a node i with degree k is infected can be expressed as $\phi_k(t) = \sum_{k'} P(k'|k) i_{k'}(t)$ which is the average over all possible degrees of k' of the probability $P(k'|k)$ that any edge of a node degree k is pointing to a node of degree k' times the probability $i_{k'}$ that the node is infected. For a random network in which the conditional probability does not depend on the originating node $P(k'|k) = k'P(k')/\langle k \rangle$,

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which can be substituted for ϕ resulting in a topology-dependent epidemic threshold given by,

$$\frac{\lambda}{\mu}=\frac{\langle k\rangle}{\langle k^2\rangle} \tag{33}$$

Following the above assumptions, subsequent results have demonstrated the link between topology and the threshold value, which governs the diffusion process on networks with different connectivity patterns [79,80]. The HMF approach cannot account for all topology-dynamical-dependent processes for instance where the timescale of transmission is short as compared to the duration of contact. As a result, more general approaches have extended HMF models where multiple occupancy of nodes can influence diffusion. For example, a general class of models based on chemical reaction-diffusion processes has been applied to sociotechnical systems where each node i can have any nonnegative number of particles N_i so that the total population of the system is $N=\sum N_i$ This approach can be applied to a network with arbitrary degree distribution where particles diffuse along edges based on a diffusion coefficient, which can account for node degree, or other attributes to be specified [2].

The above models can be further generalized into a more universal mathematical framework that aims to bridge network topology and dynamics [81]. A general model which can be mapped onto a number of dynamical models (e.g., epidemics, biochemical reactions, and population dynamics) is given by the following differential equation,

$$\frac{dx_i}{dt}=W(x_i(t))+\sum_{j=1}^{N} A_{ij}Q(x_i(t),x_j(t)) \tag{34}$$

where each node i is characterized by an activity $x_i(t)$ encoded by a nonlinear equation $W(x_i)$ represents some dynamical process such as degradation or reproduction, A_{ij} is the adjacency matrix capturing interactions between i and neighbor j, and $Q(x_i,x_j)$ describes the dynamical mechanism governing the pairwise interactions. Extensive analytical and numerical analysis have shown that regardless of the detailed structure of $W(x_i)$ and $Q(x_i,x_j)$, Eq. (34) gives distinct and finite dynamical patterns. However, a more general framework is still required to capture dynamical processes that do not take that functional form. Nevertheless, the model is able to accurately predict a complex systems' response to perturbations driven by a small number of universal characteristics indicating new avenues for theoretical and empirical work [81].

4.2. Heterogeneous Agents

Although mean field modeling techniques based on dynamical systems have been successful in describing the behavior of networked sociotechnical systems, they do not capture the interactions between decision making and network topology, which can influence diffusion dynamics [64,82]. For example, aggregate modeling techniques often show diffusion to be a smooth S-curve, because of the assumption of perfect social mixing, which can miss the high degree of variability, particularly during the early phases of diffusion [83]. Figure 4 shows empirical diffusion curves for various vehicles in UK indicating different temporal trends. This variability can be caused by multiple diffusion processes occurring simultaneously via inter market competition, and inherently random (nonutility maximizing) consumer behavior not captured by aggregate techniques [84]. This implies the need for considering how agents make strategic decisions such as, trade-offs when faced with multiple market options, and other nonlinear processes such as social contagion, or the rapid evolution of technologies (e.g., Information and Communication Technologies).

Although progress has been made in adapting mean-field approaches to better account for heterogeneity, disaggregated bottom-up approaches based on agent based simulation models are evolving rapidly. These approaches specify the behavior of individual agents, the rules of interaction with other agents, and can characterize the environment in which they interact. Simulations can be either deterministic or stochastic, where the output is typically a temporal sequence of states for all agents giving insight into the general characteristics or behavior of the modeled system. The goal is to understand the emergence of macrolevel behavior that can arise from microlevel interactions. Decarbonization of the energy system, for example, will require fundamental changes in end-use behavioral patterns, which is comprised of a multitude of microlevel behavioral interactions.

Agent-based models (ABM) can be traced back to the 1940s by Ulam and Von Neumann [86] with the introduction of cellular automaton (CA), which can be characterized by a collection of cells on a grid, where each cell is in a discrete set of states, and its behavior is updated in discrete time steps according to the state of the neighboring cells. For instance, a CA defined over a two-dimensional grid Z^2, where each grid point (i, j) is a site or node. Each site has a state $x_{i,j}(t)$ which is often a binary at time step t. The neighborhood N of a site is the collection of sites that can influence the future state $t+n$ of the given site. Based on its current state $x_{i,j}(t)$ and the current states of the sites in its neighborhood N, a function $f_{i,j}$ is used to compute the next state $x_{i,j}(t+1)$ of the site (i, j) following,

$$x_{i,j}(t+1)=f_{i,j}(\bar{x}_{i,j}(t)) \tag{35}$$

where $\bar{x}_{i,j}(t)$ denotes the tuple consisting of all the states $x_{i',j'}(t)$ with $(i',j')\in N$. The tuple consisting of the states of all the sites is the CA

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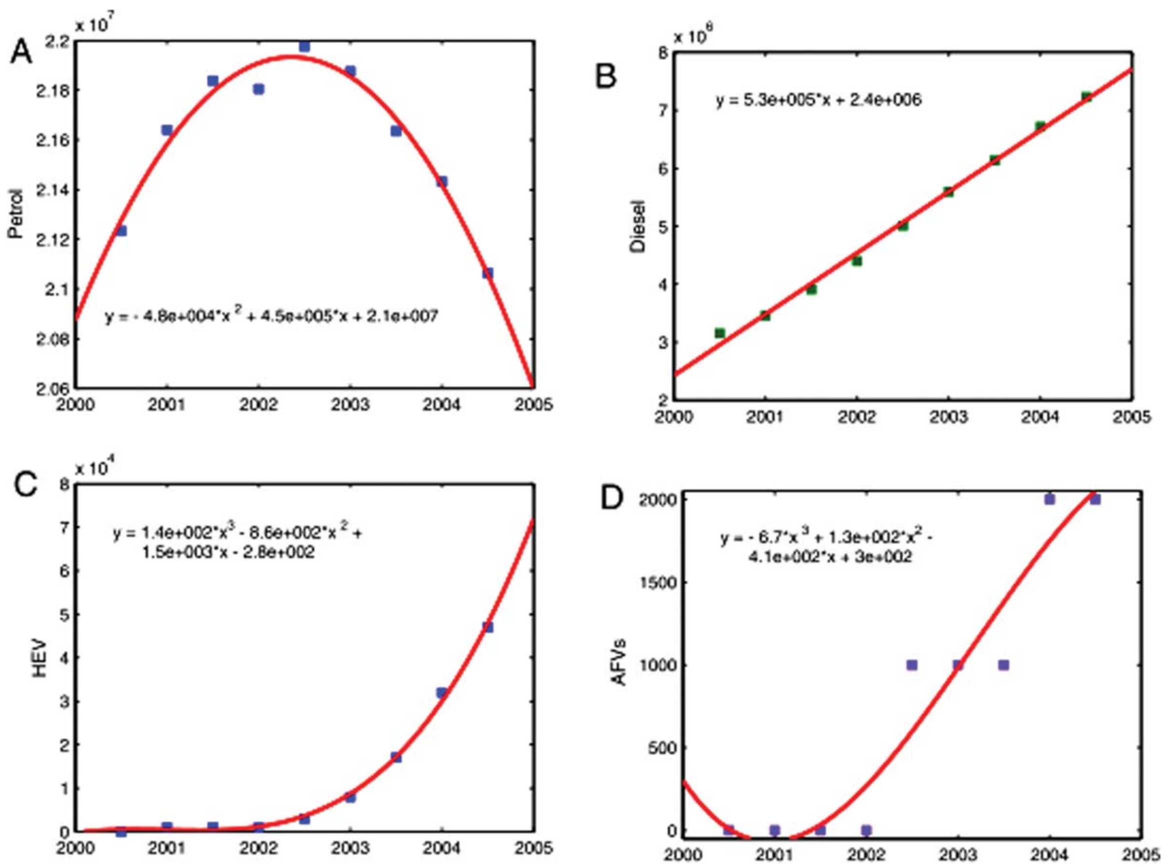
FIGURE 4



Vehicle technology diffusion in UK, 2000–2005, with fitted curves to show variability in trends. Note alternative fuel vehicle (AFV) data include electric and steam propulsion cars [85].

configuration and is denoted $x(t)=(x_{i,j}(t))i,j$. Equation (35) is used to map the configuration $x(t)$ to $x(t+1)$. The CA map ϕ sends x (t) to $x(t+1)$. The objective is to understand how configurations evolve under iteration of the map ϕ and what types of dynamical behavior can be generated [87]. Computer simulations based on ABM principles have proven effective at capturing the nondeterministic behavior arising from the interactions of large populations of heterogeneous agents [73]. The flexibility of ABMs can relax many of the assumptions implicit in conventional macrolevel diffusion models and have become increasingly used for understanding the influence of network topology on innovation diffusion [19,82,88].

4.3. Critical Discussion

There still remain important challenges to be overcome such as high computing requirements necessary to simulate large-scale sociotechnical systems, and better theoretical models of sociotechnical dynamics [2]. Additionally, there are difficulties in validating ABMs. Simulations can involve complex software constructs posing challenges for code validation. There is also no formal mathematical framework that underpins ABMs and therefore various models, and model outputs cannot be rigorously scrutinized and compared [87]. To date, the strengths of network analysis and ABM's have not been to forecast or predict. Those strengths remain more in the domain of mean-field mathematical models. Much of the work in network science has focused on describing and understanding the structural and dynamical features of real-world networks with only recent work on controlling network properties [89–91]. The strength of ABMs has been more in theoretical exploration of how network structure can be varied and shown to influence diffusion processes. In the absence of empirical data, ABMs are well suited to hypothetically assume a population with a

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degree distribution. Model parameters can then be varied to simulate different network structural properties and assess the effects on diffusion. One can also vary other properties such as the number of initial adopters or the degree of influence between adopters [83]. With improved theoretical models grounded in high-resolution time series network data, the ultimate goal is to develop network and ABM approaches able to forecast and predict the behavior and dynamics of sociotechnical systems. This is an increasingly active and important area of research with much potential to inform sustainability policy.

5. CONCLUSIONS

Sustainability transitions are governed by the macrolevel evolution of a physical system, and the microlevel processes of strategic agent behavior. These dynamics play out on physical and virtual networked topologies, which can be characterized as sociotechnical systems. We therefore critically assess modeling techniques that could be used for assessing the behavior and dynamics of such systems. This includes dynamical modeling approaches that describe the time-evolution of a system along with models that capture agent heterogeneity and strategic behavior. We also discuss the potential for complex networks and agent-based modeling techniques to simulate system and agent-level dynamics. There is much scope to apply these techniques for understanding, quantifying, and managing a transition to more sustainable sociotechnical systems. For example, understanding how network structure and dynamics influence individual adoption patterns can help us manage the subsequent large-scale diffusion of technological advancements, and behavioral practices that can either harm or benefit society and environment.

Although there is increased understanding that widespread social and physical phenomenon can be characterized by different network structures, we need to better understand how this knowledge can be leveraged to meet policy goals in the face of future environmental and societal risk and innovation.

Although important advancements have been made in network science, we are only beginning to understand the relationship between process and structure through adaptive networks [92]. Further work needs to be done in understanding how people will respond to rapid changes in connectivity patterns driven by large-scale future impacts from environmental and societal risks, or technological breakthroughs. Diffusion dynamics underpin many of these fundamental processes, therefore, improving our knowledge of diffusion pathways through complex systems in general, and networks in particular will have important implications for sustainability.

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